Turbulent Heat Transfer in Drag Reducing Fluids

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An analysis is presented which extends the analogy between energy and momentum transport for turbulent pipe flow of purely viscous fluids to include drag reducing, non-Newtonian fluids. The correlation by Meyer is used to predict friction factor and sublayer thickness for the drag reducing fluids. The use of the friction factor correlation with the heat transfer analogy makes it possible to predict heat transfer rates from simple measurements of pressure drop and flow rate for the drag reducing fluids. Some recent experimental data for two effective drag reducing fluids and for water are compared with the predicted heat transfer rates, and the mean deviation in Nusselt number is found to be +8.5% for all of the data. The heat transfer analysis predicts a reduction in Nusselt number accompanying a reduction in friction factor for a given Reynolds number and for Prandtl numbers greater than 1.

The purpose of this paper is to develop an analysis for heat transfer in turbulent pipe flow of drag reducing fluids by using the analogy between energy and momentum transport in turbulent flow and a recent correlation developed for momentum transport in drag reducing fluids. The analysis will yield a prediction of turbulent heat transfer from the knowledge only of the frictional characteristics of the fluid in turbulent pipe flow, data which are relatively easy to obtain.

Many investigators have developed successful methods of predicting heat transfer in turbulent pipe flow of purely viscous Newtonian and non-Newtonian fluids. These methods can be loosely arranged in two categories: correlations based on dimensional analysis, and expressions which assume an analogy between energy and momentum transport and use correlations for momentum transport to predict heat transfer. Since the latter method adheres more closely to a physical explanation of turbulent heat transfer, it will be used for this development.

The basis for the analogy between energy and momentum transport is found in the work of Reynolds (*1*) and Prandtl (*2*) and as discussed in references (*3*) and (*4*). This is, of course, applicable to Newtonian fluids. The Reynolds analogy does not take into account any difference between the velocity and temperature distributions across the tube, and therefore the analysis agrees well for fluids with a Prandtl number of about 1. Prandtl modified the analysis by considering the velocity distribution in the viscous sublayer, which predicts effects due to the thickness of the viscous sublayer and the Prandtl number. This relation agrees well for fluids with Prandtl numbers close to 1. A detailed investigation by Metzner and Friend (*5*), based on an analysis done by Reichardt (*6*), resulted in a modification of the Reynolds-Prandtl analogy to include an empirically determined function of Prandtl number which successfully correlates the data for Prandtl numbers from about 0.5 to 600. This relation has also been shown to predict heat transfer for purely viscous non-Newtonian fluids (*7*).

Based on these prior analyses, then, it is possible to predict the turbulent heat transfer in fully developed pipe flow for purely viscous fluids from a knowledge of the friction factor as a function of Reynolds number, the properties of the fluid, and the thickness of the viscous sublayer. Since the dimensionless thickness of the viscous sublayer $y_L u_* / \nu_w$ is constant for purely viscous fluids, the heat transfer as expressed by a dimensionless Nusselt number is a function only of the turbulent frictional characteristics and molecular properties of the fluid. Turbulent flow of drag-reducing fluids comprised of very dilute solutions of high molecular weight polymers, however, have been shown to have thickened viscous sublayers (*8* to *11*). Thus, if the concept of the analogy between energy and momentum transport is to be correctly applied to flow of drag reducing fluids, then the change in viscous sublayer thickness, as well as frictional characteristics and molecular properties, must be taken into account. Also, references *8* to *11* and *14* show no noticeable change in momentum transport in the turbulent core, as compared with Newtonian fluids. Therefore, the analogy of the type given in references *1* to *4* can be expected to hold in the turbulent core.

A correlation developed by Meyer (*12*) provides a prediction of the sublayer thickness and the friction factor as a function of Reynolds number for dilute solutions of additives which are drag reducing in turbulent flow. The two parameters which describe the deviation from purely viscous flow can be determined from simple pressure drop, flow rate measurements in a single pipe. Therefore, the combination of the friction factor and sublayer thickness correlations can be combined with the analogy between energy and momentum transport to permit prediction of heat transfer from simple pressure gradient measurements.

ANALYSIS AND CORRELATION FORMULA

From reference *3*, the Reynolds-Prandtl analogy for the turbulent core of the turbulent shear layer is given by

$$\frac{q_w}{A_w \rho C_p (T_L - T_b)} = \frac{\tau_w}{\rho(\bar{u} - u_L)} \quad (1)$$

where the subscript L indicates conditions at the edge of the viscous sublayer.

The velocity and temperature are assumed to vary linearly with distance from the wall in the viscous sublayer, and therefore the heat transfer and shear stress can be written for the sublayer as

$$\frac{q_w}{A_w C_p (T_w - T_L)} = \frac{k \tau_w}{\mu_w u_L C_p} \quad (2)$$

A combination of Equations (1) and (2) and elimination of T_L gives the heat transfer in terms of dimensionless quantities

$$N_{Nu} = \frac{(f/2) N_{Re} N_{Pr}}{\frac{u_L}{u_*} (f/2)^{1/2} (N_{Pr} - 1) + 1} \quad (3)$$

Now, Equation (3) was developed from the assumption of complete similarity between the temperature and velocity profiles. This is a good assumption for Prandtl numbers of about 1, but theoretically calculated velocity profiles by Deissler (*13*) indicate a significant difference between the profiles at high Prandtl numbers. This difference can be expressed as a function of Prandtl number only and can be put into the first term of the denominator of Equation (3), since at Prandtl numbers near 1 the first term is insignificant compared with the second, and vice versa for large Prandtl numbers. The value of the second term in the denominator can also be changed slightly to agree with the correlation for purely viscous fluids given in reference 5. Thus, Equation (3) can be written for a large range of Prandtl numbers as

$$N_{Nu} = \frac{(f/2) N_{Re} N_{Nu}}{\frac{u_L}{u_*} (f/2)^{1/2} (N_{Pr} - 1) F(N_{Pr}) + 1.2} \quad (4)$$

Reichardt has shown that $F(N_{Pr})$ for Newtonian fluids is a term which is integrated across the shear layer, most of the contribution coming from the turbulent core. Since for dilute solutions of drag reducing fluids the turbulent core is unchanged, then $F(N_{Pr})$ can be taken from a correlation for Newtonian fluids. Equation (4) is the same general form as used by Friend and Metzner (*5, 7*) to obtain $F(N_{Pr})$ for purely viscous fluids, including shear thinning fluids. They found for a large range of Prandtl numbers, in the notation of Equation (4), that

$$F(N_{Pr}) = (N_{Pr})^{-1/3} \quad (5)$$

and u_L/u_* was found to be a constant equal to 11.8.

Now u_L/u_* is a constant for purely viscous fluids, but for drag reducing fluids the dimensionless velocity has been found (*12*) to vary with shear stress such that

$$\frac{u_L}{u_*} = 5.77 \log_{10} \frac{u_L}{u_*} + 5.5 + \alpha \log_{10} \frac{u_*}{u_{*cr}} \quad (u_* \geqq u_{*cr}) \quad (6)$$

where α, the drag reduction parameter, and u_{*cr}, the critical shear stress above which drag reduction occurs, are characteristics of a particular fluid. Equation (6) is the intersection of the theoretical curves for the inner and outer similarity laws for turbulent flow for drag reducing fluids. α and u_{*cr} can be determined from the friction factor correlation given in reference *12* as

$$\frac{1}{f^{1/2}} = C_1 \log_{10} N_{Re} f^{1/2} - C_2 \quad (u_* \geqq u_{*cr}) \quad (7)$$

where

$$C_1 = 4 + \frac{\alpha}{2^{1/2}} \left(\frac{n}{2-n} \right)$$

$$C_2 = 0.394 + \frac{\alpha}{2^{1/2}} \log_{10} \left[\frac{u_{*cr} D^{\frac{n}{2-n}} 2^{\frac{n}{2(2-n)}}}{(a/\rho)^{\frac{1}{2-n}}} \right]$$

Equation (7) is written in a slightly different form than that given in reference *12* for the general case of shear thinning fluids, where a and n are the parameters of the power law purely viscous properties. Therefore, for a given fluid, α and u_{*cr} can be determined from Equation (7) with simple pressure drop flow rate measurements, and u_L/u_* can be determined as a function of u_* or N_{Re}.

Thus, the change in sublayer thickness can be taken into account in Equation (4) and can be rewritten as

$$N_{Nu} = \frac{(f/2) N_{Re} N_{Nu}}{1.02 \frac{u_L}{u_*} (f/2)^{1/2} (N_{Pr} - 1) (N_{Pr})^{-1/3} + 1.2} \quad (8)$$

It is assumed that the function $(N_{Pr})^{-1/3}$ which has been determined for purely viscous fluids is also appropriate for dilute solutions of drag reducing fluids. The validity of this assumption is indicated by the similarity between purely viscous and drag reducing fluids in the outer (or logarithmic) region of the velocity profile (*8* to *11*), which represents the largest region of the velocity profile. Similarity of turbulence intensity (fluctuation velocity normalized with u_*) through the turbulent core between purely viscous and drag reducing fluids also indicates that the structure of turbulence is not affected appreciably in the turbulent core (*14*). The factor 1.02 in Equation (8) comes about because Equation (6) gives a value of 11.6 for u_L/u_* when $\alpha = 0$, which indicates purely viscous behavior, and the ratio of the Metzner-Friend value of 11.8 to 11.6 is 1.02.

Thus, the heat transfer can be found from Equations (6) and (8), given the friction factor-Reynolds number correlation in Equation (7).

COMPARISON OF CORRELATION FORMULA WITH DATA OF PRUITT, WHITSITT, AND CRAWFORD

A recent experimental study done by Pruitt, Whitsitt, and Crawford (*15*) provides heat transfer data with

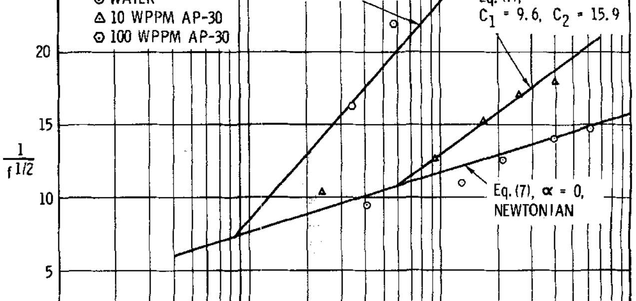

Fig. 1. Correlation of friction factor data for 10 and 100 wt. p.p.m. AP-30 and water.

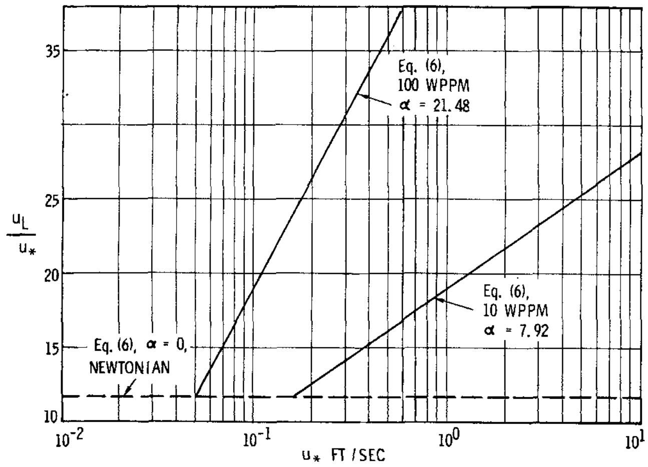

Fig. 2. Variation of velocity at the edge of the sublayer for 10 and 100 wt. p.p.m. AP-30 and for water.

which Equation (8) can be checked. A very effective drag reducing additive, Separan (AP-30), a product of the Dow Chemical Company, was used in concentrations of 10, 100, and 1,000 wt. p.p.m. in water. Both friction factor and heat transfer data were obtained for all solutions, as well as for water, in a smooth pipe of 0.50 in. I.D. The data for 10 and 100 wt. p.p.m. solutions of AP-30 were analyzed by the technique presented here; however, the 1,000 wt. p.p.m. solution was not analyzed, since velocity profile data with similarly high concentrations of effective drag reduction additives (8) indicate that the velocity profiles may not simply shift due to an increased sublayer thickness, but may also be affected in the turbulent core. If the friction factor correlation does not hold, then the heat transfer relation could not be expected to apply.

Sufficient data were available in reference *15* to apply the friction factor correlation to the 10 and 100 wt. p.p.m. solutions, as shown in Figure 1. The friction factor-Reynolds number data are plotted in accordance with the grouping of variables in Equation (7), and the correlation has been fitted to the data for each fluid. The water data show good agreement with the Newtonian relation, given by $\alpha = 0$. The best-fit straight lines drawn through the data for the polymer solutions provide values of C_1 and C_2 as shown in Figure 1. From these values, Equation (7) gives $\alpha = 7.92$ and $u_{*cr} = 0.16$ ft./sec. for 10 wt. p.p.m. and $\alpha = 23.3$ and $u_{*cr} = 0.051$ for 100 wt. p.p.m.

The variation of velocity at the edge of the sublayer, as given by Equation (6), is shown in Figure 2 for the values of α and u_{*cr} found from the data. As can be seen, the 100 wt. p.p.m. solution is much more effective, since α is relatively high and the critical value of shear stress is relatively low. The constant value of $u_L/u_* = 11.6$ for Newtonian fluids also is shown.

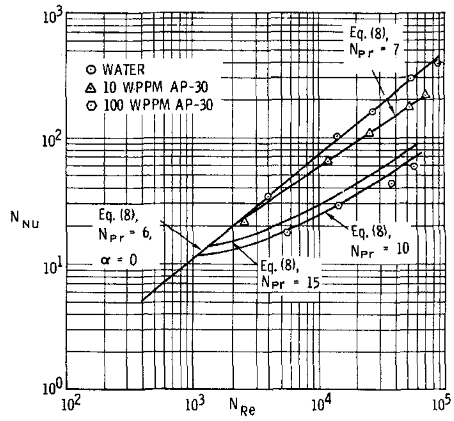

Fig. 3. Prediction of heat transfer data for 10 and 100 wt. p.p.m. AP-30 and for water.

Finally, the heat transfer data of Pruitt, Whitsitt, and Crawford are shown in Figure 3, compared with the Nusselt number expression given by Equation (8). Data for the two drag reducing solutions and for water are shown. The experimental values of the dimensionless variables are also given in Table 1. All of the data for these solutions are plotted except for a measurement at $N_{Re} = 1.06 \times 10^3$, where the flow was probably laminar. Equation (8) is plotted with the appropriate values of α and u_{*cr} for each fluid and for Prandtl numbers which are in the range of the experimental Prandtl numbers. The heat transfer expression is plotted for Prandtl numbers of 10 and 15 for the 100 wt. p.p.m. solution, since the data show a variation of N_{Pr_w} within that range.

The agreement of the data with Equation (8) is thought to be good, both in trend and in absolute accuracy. That is, the trend of the drag-reducing fluid heat transfer data to depart from the relation for Newtonian fluids at some critical value of Reynolds number is predicted quite well by Equation (8), and the percent deviation of the experimental data from the predicted values, as shown in Table 1 (calculated for each experimental Prandtl number) is considered good for heat transfer data.

Table 1. Comparison of Data of reference *15* with Predicted Nusselt Numbers

	Experimental					Calculated		
Fluid	N_{Pr}	$N_{St} \times 10^3$	$N_{Re} \times 10^{-3}$	$f/2 \times 10^3$	N_{Nu}	$\frac{u_L}{u^*}$, Equation (6)	N_{Nu}, Equation (8)	Percent Error, $\frac{N_{Nu_{calc}} - N_{Nu_{exp}}}{N_{Nu_{calc}}} \times 100$
100 wt.p.p.m.	15.89	0.20	5.6	1.88	17.8	23.9	22.48	20.82
AP-30	12.42	0.16	14.6	1.03	29.0	28.4	32.34	10.33
	9.31	0.125	38.0	0.78	44.2	34.0	55.12	19.81
	9.16	0.116	58.2	0.66	61.9	37.6	69.95	11.50
10 wt.p.p.m.	9.57	0.98	2.5	4.66	23.4	11.62	25.06	6.62
AP-30	7.33	0.75	11.9	3.10	65.4	15.2	77.16	15.26
	7.09	0.60	25.6	2.14	109	14.7	114.72	4.99
	6.94	0.48	52.6	1.72	175	17.1	182.54	4.13
	7.60	0.42	71.1	1.54	227	18.0	230.79	1.64
Water	6.15	1.448	3.88	5.54	34.5	11.62	36.00	4.17
	6.15	1.17	14.1	4.15	101.5	11.62	107.77	5.82
	6.61	0.925	26.4	3.13	161	11.62	172.30	6.56
	6.82	0.794	54.8	2.54	293	11.62	314.16	6.72
	5.74	0.77	89.5	2.34	396	11.62	443.34	10.68

The mean deviation is +8.5% for all of the data points and +10.6% for the drag reducing fluids. The mean deviation for the water data is +6.7%, compared with the mean absolute deviation for all data for Newtonian fluids given in reference *5* which is 6.8%.

It should be noted that the deviation here is all in one direction, and even better agreement could have been attained by slightly changing the empirical constant in the first term of the denominator of Equation (8). This has not been done, since the data available here for the drag reducing fluids are few compared with the data used in references *5* and *7* to determine the constants for purely viscous fluids. The purpose of this study is merely to present the technique for predicting heat transfer from simple measurements of pressure drop and flow rate and to compare the predicted values with available data. The percent deviation between the data and the predictions are small compared with the deviation from purely viscous behavior as shown by the data and predicted by this technique.

It should also be mentioned that the scatter of friction factor data shown in Figure 1 could account for the disagreement in the heat transfer prediction. The order of uncertainty is about the same in predicting skin friction from Equation (7) and heat transfer from Equation (8), which points out the need for accuracy in determining the skin friction relation in order to accurately predict heat transfer.

EXAMPLE OF DIAMETER EFFECT ON HEAT TRANSFER IN DRAG REDUCING FLUIDS

One of the most interesting aspects of the turbulent flow of drag reducing fluids is the increase in effectiveness of drag reduction with decrease in pipe diameter. This diameter effect can be predicted from the correlation by Meyer (*12*). Use of this correlation with the analogy between energy and momentum transport, as was used in developing Equation (8), will also predict a diameter effect for heat transfer in drag reducing fluids. This effect can be shown by rewriting Equation (8) in terms of the Stanton number, where

$$N_{St} \equiv \frac{N_{Nu}}{N_{Re}\, N_{Pr}} = \frac{f/2}{1.02\dfrac{u_L}{u_*}(f/2)^{1/2}(N_{Pr}-1)(N_{Pr})^{-1/3}+1.2} \qquad (9)$$

Thus the dimensionless heat transfer is expressed in Equation (9) for a given fluid and a given pipe size, as a function of friction factor only, since u_L/u_* can be expressed as

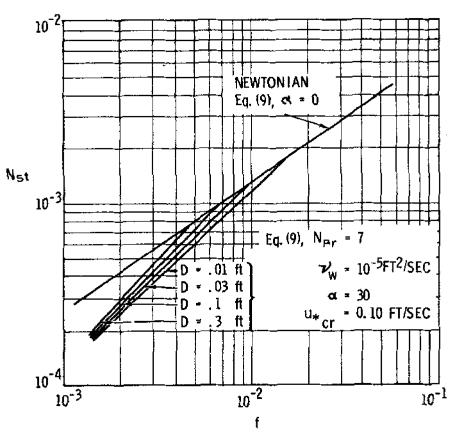

Fig. 4. Effect of pipe diameter on Stanton number for a drag reducing fluid.

a function of friction factor only. Figure 4 shows an example of this diameter effect, where the Stanton number is calculated from Equations (6), (7), and (9) as a function of f. The calculations are done for a range of diameters from 0.1 to 3 in. and for $\nu_w = 1 \times 10^{-5}$ sq.ft./sec., $\alpha = 30$, $u_{*cr} = 0.1$ ft./sec., and $N_{Pr} = 7$. As can be seen, the most effective reduction in heat transfer occurs for the smallest diameter pipe.

DISCUSSION AND SUMMARY

The available data for heat transfer in drag reducing fluids, in accordance with the results of this study, can be predicted from pressure drop, flow rate data for a given fluid, that is, a given additive and concentration, through the correlation given by Equations (6) and (7). Clearly, the limitation of this technique lies in the applicability of the friction factor correlation to particular fluids and the accuracy with which it can be determined. The accuracy has already been discussed, but something can be said concerning the generality of the correlation. The friction factor correlation has been successfully applied to several fluids over a wide range of pipe diameters (*10, 12*); however, if there is a question of the applicability of the correlation, it can be tested by performing the pressure drop flow rate experiments in pipes of two different diameters for the fluid of interest. This would be recommended if the fluid properties or range of diameters exceeds the range of results presented previously. It is probable that a lower limit of pipe diameter exists below which the correlation cannot be applied, since the wall shear stress should not be reduced below that for laminar flow. An investigation of this limit is being made at this time.

Another possible limitation to the technique is the range of fluid temperatures over which the parameters α and u_{*cr} can be used, since the results presented here are for small temperature variations. This is believed to be a no more serious problem than that of using the Newtonian heat transfer relation for large temperature difference, however. As noted in reference *5*, the analogy relations assume reasonably small temperature differences. It should also be noted that the purely viscous properties of polymer solutions at greatly elevated temperatures are not well known, and the usual approach is to use the same corrections as for water when the temperature differences are small. Large temperature differences could also be a source of error in that regard, but the results of this study show that there is not a problem when the differences are small.

A point that should be made is that the agreement shown in Figure 3 indicates that the thermal conductivity for the AP-30 is about the same as for water, since that value was used in the experimental data reduction. This question was raised in reference *15*.

A more general result of the comparisons between the heat transfer analysis and the data is the additional confirmation that the viscous sublayer is thickened for drag reducing fluids. This is indicated, since, in accordance with Equation (8), if the term denoting sublayer thickening is left constant at the Newtonian value, a large deviation between predicted and observed heat transfer rate would result.

It should also be noted that the analysis for drag reducing liquids, where the Prandtl number is greater than 1, predicts that a reduction in friction factor is accompanied by a reduction in Nusselt number, at a given Reynolds number.

ADDENDUM

During the review of this paper the author obtained an additional set of heat transfer data for turbulent pipe flow

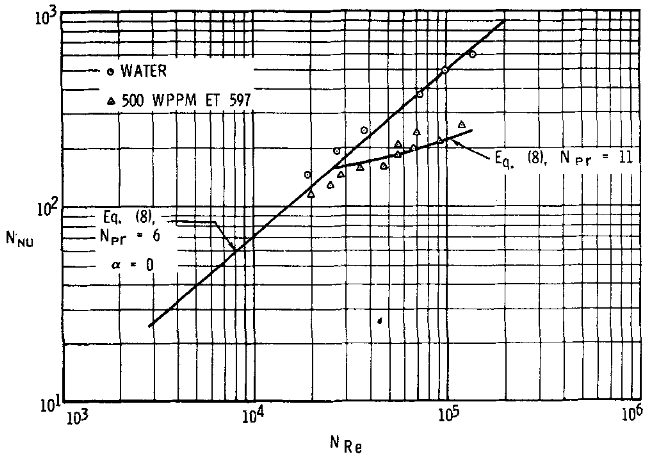

Fig. 5. Prediction of heat transfer data for 500 wt. p.p.m. ET 597 and for water.

of drag reducing fluids. Since the purpose of this paper is to compare available experimental data with the correlation given by Equation (8), it is appropriate to add this comparison to those already presented.

Heat transfer data were obtained by Gupta (*16*) for water and a 500 wt. p.p.m. solution of ET-597 in water under conditions of turbulent pipe flow. ET-597 is a water soluble, partially hydrolysed polyacrylamide of high molecular weight, a product of the Dow Chemical Company. The 500 wt. p.p.m. solution demonstrated a considerable drag reduction at N_{Re} between 10^4 and 10^5. Solutions of 100 and 4,500 wt. p.p.m. were also tested, but the former was not significantly drag reducing and the latter did not show a clear transition to turbulent flow. Therefore, the 500 wt. p.p.m. solution was chosen for comparison as the only fluid which fits the conditions of the analysis under discussion.

The data were analyzed in a similar manner to those presented in Figures 1 and 2. The heat transfer data given in reference *16* were obtained with much higher temperature differentials than those given in reference *15*, and the experimental values of k and u_w were evaluated at the bulk temperature of the fluid in reference *16*. This is consistent with the previous correlations for purely viscous fluids (*5, 7*). In reference *15* the properties were evaluated at the wall temperature, but the temperature differentials were small enough that the effect was negligible. This is confirmed by the data for water given in Figure 3.

The correlation constants determined for the 500 wt. p.p.m. ET 597 solution are $\alpha = 26.5$ and $u_{*cr} = 0.31$ ft./sec. The correlation for the drag reducing fluid and for water are shown in Figure 5, compared with the experimental data. The correlation was computed for typical values of N_{Pr} of 6 and 11 for water and ET 597, respectively. The actual values of N_{Pr} varied from 4.86 to 6.46 for the former and from 10.88 to 11.50 for the latter. The predicted values of N_{Nu} in accordance with Equation (8) were also calculated for each experimental condition. The mean absolute deviation from the predicted values for all of the data is 11.39% and 9.89% for the drag reducing fluid data. The mean absolute deviation for the water is 14.11%. This is thought to be reasonable agreement with predicted values and is comparable to the agreement found for purely viscous fluids in references *5* and *7*.

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NOTATION

A_w = area of wall
a = coefficient of power law viscosity relation
C_p = heat capacity at constant pressure
D = diameter of pipe
F = function of Prandtl number
f = friction factor
h = heat transfer coefficient, $q_w/A_w(T_w - T_b)$
k = thermal conductivity of fluid
n = exponent of power-law viscosity relation
N_{Nu} = Nusselt number, $h\,D/k$
N_{Re} = Reynolds number based on viscosity at the wall
N_{Pr} = Prandtl number based on viscosity at the wall
N_{St} = Stanton number
q_w = heat transfer rate at the wall
T = temperature
u = local velocity in the axial direction
u_* = shear velocity, $(\tau_w/\rho)^{1/2}$
u_{*cr} = critical shear velocity, above which drag reduction occurs
$\bar{u}$ = bulk mean velocity
y = distance from wall

Greek Letters

α = drag reduction parameter, defined by Equations (6) or (7)
τ_w = shear stress at the wall
ρ = fluid density
μ = fluid viscosity
ν = fluid kinematic viscosity

Subscripts

b = bulk property
L = evaluated at the edge of the viscous sublayer
w = evaluated at the wall

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